Class XII Session 2025-26
Subject - Applied Mathematics
Sample Question Paper - 7

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory.
2. Section - A carries 20 marks weightage, Section - B carries 10 marks weightage, Section - C carries 18 marks weightage, Section - D carries 20 marks weightage and Section - E carries 3 case-based with total weightage of 12 marks.
3. **Section – A:** It comprises of 20 MCQs of 1 mark each.
4. **Section – B:** It comprises of 5 VSA type questions of 2 marks each.
5. **Section – C:** It comprises of 6 SA type of questions of 3 marks each.
6. **Section – D:** It comprises of 4 LA type of questions of 5 marks each.
7. **Section – E:** It has 3 case studies. Each case study comprises of 3 case-based questions, where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.
8. Internal choice is provided in 2 questions in Section - B, 2 questions in Section – C, 2 questions in Section - D. You have to attempt only one of the alternatives in all such questions.

Section A

1. If A = $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then the value of |adj A| is **[1]**

a) a^9 b) a^2

c) a^6 d) a^{27}

2. If the calculated value of $|t| < t_v(\alpha)$, then the null hypothesis is: **[1]**

a) cannot be determined b) accepted

c) neither accepted nor rejected d) rejected

3. Assume that Shyam holds a perpetual bond that generates an annual payment of ₹500 each year. He believes that the borrower is creditworthy and that an 8% interest rate will be suitable for this bond. The present value of this perpetuity is **[1]**

a) ₹6250 b) ₹6520

c) ₹5620 d) ₹2650

4. The objective function of an LPP is **[1]**

a) a constrain

b) a relation between the variables

c) a function to be optimized

d) a function to be not optimized

5. . Let A be a square matrix of order 2×2, then |KA| is equal to: **[1]**

a) K^3 |A|

b) K^2 |A|

c) K |A|

d) 2K |A|

6. A random variable X takes the values 0, 1, 2, 3 and its mean is 1.3. If P(X = 3) = 2 P(X = 1) and P(X = 2) = 0.3, then P(X = 0) is: **[1]**

a) 0.1

b) 0.4

c) 0.3

d) 0.2

7. A coin is tossed 4 times. The probability that at least one head turns up, is **[1]**

a) $\frac{1}{16}$

b) $\frac{15}{16}$

c) $\frac{2}{16}$

d) $\frac{14}{16}$

8. The order and the degree of the differential equation of the family of curves given by $y = Ax + A^3$, where A is arbitrary constant, are **[1]**

a) 1, 2

b) 1, 1

c) 1, 3

d) 2, 3

9. A pipe can empty $\frac{5}{6}$ of a cistern in 20 minutes. What part of cistern will be emptied in 9 minutes **[1]**

a) $\frac{4}{5}$

b) $\frac{5}{8}$

c) $\frac{3}{5}$

d) $\frac{3}{8}$

10. In a game, A can give B 25 points, A can give C 40 points and B can give C 20 points. How many points make the game? **[1]**

a) 100

b) 120

c) 80

d) 150

11. The smallest non-negative integer congruent to 2796 (mod 7) is **[1]**

a) 2

b) 3

c) 1

d) 5

12. If $|2x + 3| < 7$, $x \in R$, then **[1]**

a) $x \in (-\infty, -5) \cup (2, \infty)$

b) $x \in (-5, 2]$

c) $x \in (-\infty, -5] \cup [2, \infty)$

d) $x \in (-5, 2)$

13. A runs $1\frac{2}{3}$ times as fast as B. If A gives B a start of 80 m, how far must the winning post be so that A and B may reach it at the same time? **[1]**

a) 100 m

b) 240 m

c) 200 m

d) 120 m

14. The position of points O(0, 0) and P(2, -2) in the region of graph of in equation $2x - 3y < 5$ will be **[1]**

a) O outside and P inside

b) O and P both inside

c) O and P both outside

d) O inside and P outside

15. If the marginal revenue function of a commodity is MR = $2x - 9x^2$, then the revenue function is **[1]**

a) 2 - 18x

b) $2x^2 - 9x^3$

c) $18 + x^2 - 3x^3$

d) $x^2 - 3x^3$

16. For a student's t-test, the test statistic t is given by: **[1]**

a) $t = \frac{\overline{x}}{s}$

b) $t = \frac{\overline{x}-\mu}{\frac{s}{\sqrt{n-1}}}$

c) $t = \overline{x} - \mu$

d) $t = \frac{\overline{x}-\mu}{s}$

17. $\int_a^b \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a+b-x}} dx$ is equal to **[1]**

a) π

b) $b - a$

c) $\frac{\pi}{2}$

d) $\frac{1}{2}(b-a)$

18. For the given values 15, 23, 28, 36, 41, 46, the 3-yearly moving averages are: **[1]**

a) 22, 29, 35, 41

b) 22, 28, 35, 41

c) 24, 29, 35, 41

d) 24, 28, 35, 41

19. **Assertion (A):** If A = $\begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$, then A^{-1} does not exist. **[1]**

Reason (R): On using elementary column operations $C_2 \to C_2 - 2C_1$ in the following matrix equation

$\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$, we have $\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The function f(x) = sin x decreases on the interval $(0, \frac{\pi}{2})$. **[1]**

Reason (R): The function f(x) = cos x decreases on the interval $(0, \frac{\pi}{2})$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. The revenues of company over a period are given as follows: **[2]**

Year	**2015**	**2016**	**2017**	**2018**
Revenue (in thousands ₹)	100	115	150	200

Calculate CAGR over the 3-year period spanning the end of 2015 to the end of 2018. [Given: $(2)^{\frac{1}{3}}$ = 1.26]

22. 10 years ago, Mr Mehra set-up a sinking fund to save for his daughter's higher studies. At the end of 10 years, he received an amount of ₹ 10,21,760. What amount did he put in the sinking fund at the end of every 6 months for the tenure, which paid him 5 % p.a. compounded semiannually? [Use $(1.025)^{20}$ = 1.6386] **[2]**

OR

Find the declared rate of return compounded semiannually which is equivalent to 6% effective rate of return [Use

$(1.06)^{\frac{1}{2}}$ = 1.0296]

23. Evaluate the definite integral: **[2]**

$$\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}}dx$$

24. Solve the matrix equation: $\begin{bmatrix} 2x & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ 8 \end{bmatrix} = 0$ **[2]**

OR

If A = $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find the value of λ so that $A^2 = \lambda A - 2I$. Hence, find A^{-1}.

25. A container contains 50 litres of milk. From this container 10 litres of milk was taken out and replaced by water. This process is repeated two more times. How much milk is now left in the container? **[2]**

Section C

26. If $\sqrt{1-x^2} + \sqrt{1-y^2} = 4(x - y)$, then show that $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$. **[3]**

OR

Solve the initial value problem: $x\frac{dy}{dx} + y = x \log x$, $y(1) = \frac{1}{4}$

27. Mr. M borrowed ₹10,00,000 from a bank to purchase a house and decided to repay by monthly equal instalments in 10 years. The bank charges interest at 9% compounded annually. The bank calculated his EMI as ₹12,668. Find the principal and interest paid in first year. [Given $a_{108\rceil 0.0075}$ = 73.83916] **[3]**

28. A company has approximated the marginal cost and marginal revenue functions for one of its products by MC = $81 - 16x + x^2$ and MR = $20x - 2x^2$ respectively. Determine the profit-maximizing output and the total profit at the optimal output, assuming fixed cost as zero. **[3]**

29. An urn contains 5 white, 7 red and 8 black balls. If four balls are drawn one by one with replacement, what is the probability that **[3]**

i. all are white?
ii. only 3 are white?
iii. none is white?
iv. at least three are white?

OR

The probability that a student entering a university will graduate is 0.4. Find the probability that out of 3 students of the university:

i. none will graduate
ii. only one will graduate
iii. all will graduate.

30. The production of soft drink company in thousands of litres during each month of a year is as follows: **[3]**

Jan	Feb	March	April	May	June	July	August	Sept.	Oct.	Nov.	Dec.
1.2	0.8	1.4	1.6	1.8	2.4	2.6	3.0	3.6	2.8	1.9	3.4

Calculate the five monthly moving averages and show these moving averages on a graph.

31. A machinist is making engine parts with axle diameter of 0.7 inch. A random sample of 10 parts shows mean diameter 0.742 inch with a standard deviation of 0.04 inch. On the basis of this sample, would you say that the work is inferior? (Given t_9 (0.05) = 2.262) **[3]**

Section D

32. Solve the following LPP graphically: **[5]**

Maximize Z = 5x + 3y

Subject to $3x + 5y \leq 15$

$5x + 2y \leq 10$

and, $x, y \geq 0$

OR

Solve the following LPP graphically:

Minimize and Maximize Z = 5x + 2y

Subject to

$-2x - 3y \leq -6$

$x - 2y \leq 2$

$3x + 2y \leq 12$

$-3x + 2y \leq 3$

$x, y \geq 0$

33. A man take twice as long as to row a distance against the stream to row the same distance in the direction of stream. Find the ratio of speed of man in still water to the speed of stream **[5]**

34. If 1% of the electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs, the number of defective bulbs will be 0, 1, 2, 3, 4, 5 respectively. Use recurrence relation of Poisson distribution. Also find the probability that **[5]**

i. 3 or more
ii. between 1 and 3, and
iii. less than or equal to 2 bulbs will be defective.

OR

Phone calls arrive at the rate of 48 per hour at the reservation desk for Indian Airlines.

i. Compute the probability of receiving three calls in a 5 minutes interval of time.
ii. Compute the probability of receiving exactly 10 calls in 15 minutes.
iii. Suppose no calls are currently on hold. If the agent takes 5 minutes to complete the current call, how many do you expect to be waiting by that time? What is the probability that none will be waiting?
iv. If no calls are currently being processed, what is the probability that the agent can take 3 minutes for personal time without being interrupted by a call?

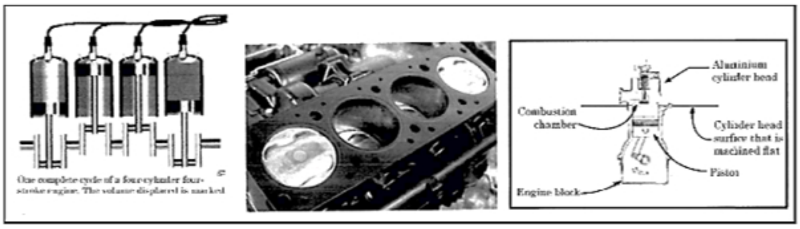
35. A recent accounting graduate opened a new business and installed a computer system that costs ₹ 45,200. The computer system will be depreciated linearly over 3 years and will have a scrap value of ₹ 0. **[5]**

i. What is the rate of depreciation?
ii. Give a linear equation that describes the computer system's book value at the end of t^{th} year, where $0 \leq t \leq 3$.
iii. What will be the computer system's book value at the end of the first year and a half?

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore



The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area 75π cm^2.

(a) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r.

(b) Find $\frac{dV}{dr}$.

(c) Find the radius of cylinder when its volume is maximum.

OR

For maximum volume, h > r. State true or false and justify.

37. **Read the text carefully and answer the questions:** **[4]**

Loans are an integral part of our lives today. We take loans for a specific purpose - for buying a home, or a car, or sending kids abroad for education - loans help us achieve some significant life goals. That said, when we talk about loans, the word “EMI", eventually crops up because the amount we borrow has to be returned to the lender with interest.

Suppose a person borrows ₹1 lakh for one year at the fixed rate of 9.5 percent per annum with a monthly rest. In this case, the EMI for the borrower for 12 months works out to approximately ₹8,768.

Example:

In year 2000, Mr. Tanwar took a home loan of ₹3000000 from State Bank of India at 7.5% p.a. compounded monthly for 20 years.

(a) Find the equated monthly installment paid by Mr. Tanwar.

(b) Find interest paid by Mr. Tanwar in 150th payment.

(c) Find Principal paid by Mr. Tanwar in 150th payment.

OR

Find principal outstanding at the beginning of 193th month.

38. If A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of linear equations **[4]**

x + 2y + z = 4, -x + y + z = 0, x - 3y + z = 2

OR

If A = $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, show that $A^{-1} = A^3$.

Solution

Section A

1.

   (c) a^6

   Explanation:

   A = $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

   $|A| = a^3$

   $|adj\ A| = |A|^{3-1} = |A|$

   $|adj\ A| = (a^3)^2 = a^6$

2.

   (b) accepted

   Explanation:

   accepted

3. **(a)** ₹6250

   Explanation:

   PV of perpetuity

   $= \frac{Annual Payment/Cash flow}{Interest rate/yield}$

   $= \frac{500}{8}$

   $= \frac{500}{0.08}$

   = ₹6250

4.

   (c) a function to be optimized

   Explanation:

   A Linear programming problem is a linear function (also known as an objective function) subjected to certain constraints for which we need to find an optimal solution (i.e. either a maximum/minimum value) depending on the requirement of the problem.

   From the above definition, we can clearly say that the Linear programming problem's objective is to either maximize/minimize a given objective function, which means to optimize a function to get an optimum solution.

5.

   (b) K^2 |A|

   Explanation:

   K^2 |A|

6.

   (b) 0.4

   Explanation:

   Let P(X = 0) = m

   P(X = 1) = k

   Now,

   P(X = 3) = 2k

x_i	p_i	$p_i x_i$

0	m	0
1	k	k
2	0.3	0.6
3	2k	6k

Mean = $\sum p_i x_i$

0 + k + 0.6 + 6k = 1.3

$\Rightarrow$ 7k = 1.3 - 0.6

$\Rightarrow$ k = $\frac{0.7}{7}$ = 0.1

We know that the sum of probabilities in a probability distribution is always 1.

$\therefore$ P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 1

$\Rightarrow$ m + 0.1 + 0.3 + 0.2 = 1

$\Rightarrow$ m + 0.6 = 1

$\Rightarrow$ m = 0.4

7.

(b) $\frac{15}{16}$

Explanation:

n = 4, p = q = $\frac{1}{2}$

P(X $\geq$ 1) = 1 - P(X = 0)

P(X $\geq 1) = 1 - \left(\frac{1}{2}\right)^4$

P(X $\geq 1) = \frac{15}{16}$

8.

(c) 1, 3

Explanation:

y = Ax + A^3 $\Rightarrow$ $\frac{dy}{dx}$ = A

$\therefore$ The differential equation of family of curves is

y = x$\left(\frac{dy}{dx}\right)$ + $\left(\frac{dy}{dx}\right)^3$

$\therefore$ Order = 1, degree = 3

9.

(d) $\frac{3}{8}$

Explanation:

$\because$ In 20 minutes $\frac{5}{6}$ part of cistern is emptied.

$\therefore$ In 1 minutes $\frac{\frac{5}{6}}{20}$ part of cistern is emptied

$\therefore$ In 9 minutes $\frac{5}{6 \times 20} \times 9$ part of cistern is emptied

Hence, $\frac{3}{8}$ part of cistern will be emptied in 9 minutes.

10. **(a)** 100

Explanation:

Let P points make the game

When A score P points, B score (P - 25) point and C score (P - 40) points

Now, When B scores P points, C scores (P - 20) point

When B score, (P - 25), C score = $\frac{(P-20)}{P} \times (P-25)$

A/C to ques:

$\frac{(P-20)(P-25)}{P}$ = P - 40

p^2 - 20p - 25p + 500 = p^2 - 40p

-45p + 40p = -500

-5p = -500

p = 100

Hence, 100 points make the game.

11.

(b) 3

Explanation:

Let x be the smallest integer that satisfies 2796 (mod 7)

$\Rightarrow \frac{2796}{7} = x$

$\Rightarrow x = 3$

12.

(d) $x \in (-5, 2)$

Explanation:

$|2x + 3| < 7 \Rightarrow -7 < 2x + 3 < 7$

$\Rightarrow -7 - 3 < 2x < 7 - 3 \Rightarrow -10 < 2x < 4$

$\Rightarrow -5 < x < 2$

$\therefore x \in (-5, 2)$

13.

(c) 200 m

Explanation:

Let the winning post be at a distance of x m, then distance covered by B = (x - 80) m

Given A's speed $= \frac{5}{3}$ B's speed

$\Rightarrow \frac{\text{A's speed}}{\text{B's speed}} = \frac{5}{3} \Rightarrow \frac{x \times t}{(x-80) \times t} = \frac{5}{3}$

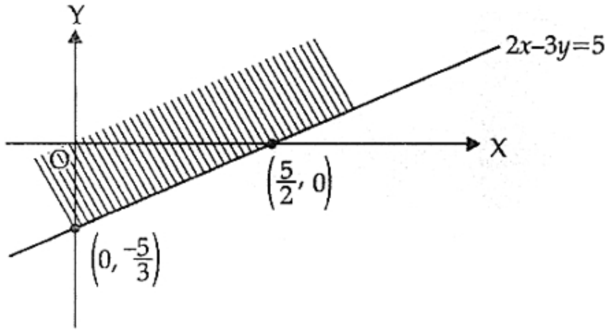
where t = time taken to reach the winning post

$\Rightarrow 5x - 400 = 3x \Rightarrow x = 200$ m

14.

(d) O inside and P outside

Explanation:



15.

(d) $x^2 - 3x^3$

Explanation:

Given MR = $2x - 9x^2$

$\therefore R(x) = \int (2x - 9x^2)\, dx$

$\Rightarrow R(x) = x^2 - 3x^3 + k$

We know that when x = 0, R(x) = 0

$\Rightarrow 0 - 0 + k = 0 \Rightarrow k = 0$

$\therefore R(x) = x^2 - 3x^3$

16.

(c) $t = \overline{x} - \mu$

Explanation:

$t = \overline{x} - \mu$

17.

(d) $\frac{1}{2}(b-a)$

Explanation:

Let $I = \int_a^b \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a+b-x}} dx$ ...(i)

$I = \int_a^b \frac{\sqrt{a+b-x}}{\sqrt{a+b-x}+\sqrt{a+b-(a+b-x)}} dx$ (by property P_3)

$\Rightarrow \quad I = \int_a^b \frac{\sqrt{a+b-x}}{\sqrt{a+b-x}+\sqrt{x}} dx$ ...(ii)

Adding (i) and (ii), we get

$2I = \int_a^b 1 dx = [x]_a^b = b - a$

$\Rightarrow I = \frac{b-a}{2}$

18. **(a)** 22, 29, 35, 41

Explanation:

22, 29, 35, 41

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Assertion: We have, A = IA

i.e., $\begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{5} \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{10} & 0 \\ 0 & 1 \end{bmatrix} A$ [applying $R_1 \rightarrow \frac{1}{10} R_1$]

$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{5} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{10} & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$ [applying $R_2 \rightarrow R_2 + 5R$]

We have all zeroes in the second row of the left hand side matrix of above equation. Therefore, A^{-1} does not exist.

Reason: The given matrix equation is $\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

$\because$ The column transformation $C_2 \rightarrow C_2 - 2C_1$ is applied.

$\therefore$ This transformation is applied on LHS and on second matrix of RHS.

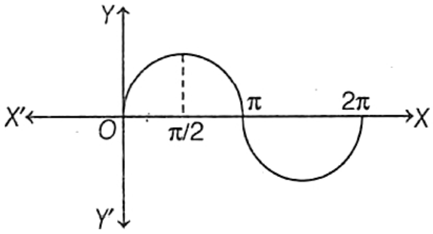
Thus, we have $\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$.

20.

(d) A is false but R is true.

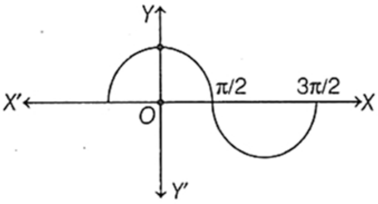
Explanation:

Assertion: Given, function f(x) = sin x



From the graph of sin x, we observe that f(x) increases on the interval (0, $\frac{\pi}{2}$).

Reason: Given function is f(x) = cos x.



From the graph of cos x, we observe that, f(x) decreases on the interval (0, $\frac{\pi}{2}$).

Hence, Assertion is false and Reason is true.

Section B

21. The Compound Annual Growth Rate (CAGR) can be calculated using the following formula:

CAGR = $\left(\frac{EV}{BV}\right)^{\frac{1}{n}}$ - 1

Now, plug these values into the formula:

CAGR = $\left(\frac{200,000}{100,000}\right)^{\frac{1}{3}}$ - 1

CAGR = $(2)^{\frac{1}{3}}$ - 1

1.26 = $(2)^{\frac{1}{3}}$ - 1

Now, let's solve for $2^{\frac{1}{3}}$:

$2^{\frac{1}{3}}$ = 1.26 + 1

$2^{\frac{1}{3}}$ = 2.26

Now, raise both sides to the power of 3 to isolate 2:

2 = $(2.26)^3$

Now, calculate 2.26^3

2 ≈ 12.1657

So, the approximate CAGR over the 3-year period spanning the end of 2015 to the end of 2018 is 12.1657%.

22. Given A = ₹ 1021760, r = 5 % p.a. = 2.5 % per year

⇒ i = $\frac{2.5}{100}$ = 0.025 and n = 10 years = 20 half years, R = ?

Using formula $A = R\left[\frac{(1+i)^n-1}{i}\right]$

⇒ 1021760 = R $\left[\frac{(1.025)^{20}-1}{0.025}\right]$

⇒ $1021760 = R\left[\frac{1.6386-1}{0.025}\right]$

⇒ R = $\frac{1021760\times0.025}{0.6386}$ ⇒ R = ₹ 40000

OR

Let declared rate of interest be r % p.a. compounded half yearly.

Given effective rate of return (per rupee) = $\frac{6}{100}$ = 0.06 (per-rupee), p = 2 half years.

∴ 0.06 = $\left(1+\frac{r}{200}\right)^2$ - 1

⇒ $\left(1+\frac{r}{200}\right)^2$ = 1.06 ⇒ 1 + $\frac{r}{200}$ = $(1.06)^{\frac{1}{2}}$

⇒ 1 + $\frac{r}{200}$ = 1.0296 ⇒ $\frac{r}{200}$ = 0.0296

⇒ r = 0.0296 × 200 ⇒ r = 5.92

Hence, the declared rate of return = 5.92%

23. Let I = $\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}}dx$ ...(1)

Then, by using $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get

I = $\int_1^2 \frac{\sqrt{1+2-x}}{\sqrt{3-(1+2-x)}+\sqrt{1+2-x}}dx = \int_1^2 \frac{\sqrt{3-x}}{\sqrt{x}+\sqrt{3-x}}dx$ ...(2)

On adding (1) and (2), we get

$2I = \int_1^2 \frac{\sqrt{x}+\sqrt{3-x}}{\sqrt{x}+\sqrt{3-x}} dx = \int_1^2 1 dx = [x]_1^2$ = 2 - 1 = 1

$\Rightarrow I = \frac{1}{2}$

24. Given, $\begin{bmatrix} 2x & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ 8 \end{bmatrix} = 0$

$\Rightarrow [2x \times 1 + 3 \times (-3) \quad 2x \times 2 + 0] \begin{bmatrix} x \\ 8 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 2x - 9 & 4x \end{bmatrix} \begin{bmatrix} x \\ 8 \end{bmatrix} = 0$

Again, multiply these two LHS matrices, we get

$\Rightarrow [(2x - 9) \times x + 4x \times 8] = 0$

$\Rightarrow 2x^2 - 9x + 32x = 0$

$\Rightarrow 2x^2 - 23x$. This is form of quadratic equation, we will solve this, we get

$\Rightarrow x(2x - 23) = 0$

$\Rightarrow x = 0$ or $2x - 23 = 0$

This gives, $x = 0$ or $x = \frac{23}{2}$ is the required solution of the matrices.

OR

Given: $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$

Now, $A^2 = \lambda A - 2I$

$\Rightarrow \lambda A = A^2 + 2I$

$= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$= \lambda \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$= \begin{bmatrix} 3\lambda & -2\lambda \\ 4\lambda & -2\lambda \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$= 3\lambda = 3$ or $\lambda = 1$

So, $A^2 = A - 2I$

Multiply by A^{-1} both sides

$= A.A.\ A^{-1} = A.\ A^{-1} - 2I.\ A^{-1} = 0$

$= 2A^{-1} = I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix}$

Hence, $A^{-1} = \frac{1}{2} \cdot \begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix}$

25. Given x = 50 litres, y = 10 litres, n = 3.

So, amount of milk left $= x\left(1 - \frac{y}{x}\right)^n = 50\left(1 - \frac{10}{50}\right)^3$

$= 50 \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$

$= \frac{128}{5} = 25.6$ litres

Section C

26. $\sqrt{1 - x^2} + \sqrt{1 - y^2} = 4(x - y)$

put $x = \sin\theta$, $y = \sin\theta$

$\theta = \sin^{-1}x$ $\phi = \sin^{-1}y$

$\sqrt{1 - \sin^2\theta} + \sqrt{1 - \sin^2\phi} = 4(\sin\theta - \sin\phi)$

$\cos\theta + \cos\phi = 4\sin\theta - \sin\phi$

$2\cos\left(\frac{\theta+\phi}{2}\right)\cos\frac{\theta-\phi}{2} = 2 \cdot 4\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$

$\frac{\cos\theta-\phi}{2} = 4 \cdot \sin\frac{\theta-\phi}{2}$

$\frac{\cos\left(\frac{\theta-\phi}{2}\right)}{\sin\left(\frac{\theta-\phi}{2}\right)} = 4$

$\frac{\theta-\phi}{2}$ = cot^{-1}4

θ - ϕ = 2 cot^{-1}4

sin^{-1}x - sin^{-1}y = 2 cot^{-1}4

diff. w.r.t. x we get

$\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\cdot\frac{dy}{dx}=0$

$\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

Hence proved

OR

We have, x$\frac{dy}{dx}$ + y = x log x

$\Rightarrow \frac{dy}{dx}+\frac{1}{x}y$ = log x ...(i)

This is linear differential equation of the form $\frac{dy}{dx}$ + Py = Q with P = $\frac{1}{x}$ and Q = log x

$\therefore$ I.F. = $e^{\int\frac{1}{x}dx}$ = e$^{\log x}$ = x [$\because$ x > 0]

Multiplying both sides of (i) by I.F. = x, we get

x$\frac{dy}{dx}$ + y = x log x

Integrating with respect to x, we ge

yx = $\int \underset{II}{x} \underset{I}{\log x}\, dx$ [Using: y (I.F.) = $\int$ Q (I.F.) dx + C]

$\Rightarrow$ yx = $\frac{x^2}{2}$(log x)$\frac{1}{2}$ $\int$ x dx

$\Rightarrow$ yx = $\frac{x^2}{2}$(log x) - $\frac{x^2}{4}$ + C ...(ii)

It is given that y (1) = $\frac{1}{4}$ i.e. y = $\frac{1}{4}$ where x = 1. Putting x = 1 and y = $\frac{1}{4}$ in (ii), we get

$\frac{1}{4}$ = 0 -$\frac{1}{4}$ + C $\Rightarrow$ C = $\frac{1}{2}$

Putting C = $\frac{1}{2}$ in (ii), we get

xy = $\frac{x^2}{2}$(log x) -$\frac{x^2}{2}$ + $\frac{1}{2}$ $\Rightarrow$ y = $\frac{1}{2}$x log x - $\frac{x}{4}+\frac{1}{2x}$

Hence, y = $\frac{1}{2}$ x log x - $\frac{x}{4}+\frac{1}{2x}$ is the solution of the given differential equation.

27. Principal left unpaid after one year (12 payments)

= present value of remaining 108 payments = $Ra_{n\rceil i}$

Where R = 12,668, n = 108 and i = $\frac{0.09}{12}$ = 0.0075

$= 12,668 \times a_{108\rceil 0.0075}$

= 12,668 $\times$ 73.83916 = ₹ 9,35,395

Principal paid during first year = 10,00,000 - 9,35,395 = ₹64,605

Interest paid during first year

= (12,668 $\times$ 12) - 64,605

= ₹ 87,651

28. Let P be the profit function. Then,

$\frac{dP}{dx}$ = MR - MC

$\Rightarrow \frac{dP}{dx}$ = (20x - 2x^2) - (81 - 16x + x^2)

$\Rightarrow \frac{dP}{dx}$ = -81 + 36x - 3x^2 ... (i)

$\Rightarrow \frac{dP}{dx}$ = -81 + 36x - 3x^2 and $\frac{d^2P}{dx^2}$ = 36 - 6x

For P to be maximum, we must have

$\frac{dP}{dx}$ = 0

$\Rightarrow$ -81 + 36x - 3x^2 = 0

$\Rightarrow$ -3(x^2 - 12x + 27) = 0

$\Rightarrow$ -3(x - 3) (x - 9) = 0

$\Rightarrow$ x = 3 or, x = 9

We find that

$\left(\frac{d^2P}{dx^2}\right)_{x=9}$ = 36 - 6 $\times$ 9 = -18 < 0

Thus, the output x = 9 gives maximum profit.

From (i), we obtain

$\frac{dP}{dx}$ = -81 + 36x - 3x^2

Integrating both sides, we obtain

P = -81x + 18x^2 - x^3 + k ... (ii)

When x = 0, fixed cost = 0 i.e. there is no profit. So, putting x = 0, P = 0 in (ii), we obtain

P = -81x + 18x^2 - x^3

Putting x = 9, we obtain

P = - 81 $\times$ 9 + 18 $\times$ 9^2 - 9^3 = 0

Hence, there is no profit when 9 items are produced.

29. Let p denote the probability of drawing a white ball from an urn containing 5 white, 7 red and 8 black balls. Then,

$p = \frac{^5C_1}{^{20}C_1} = \frac{5}{20} = \frac{1}{4}$. So, $q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$

Let X be a random variable denoting the number of white balls in 4 draws with replacement. Then, X is a binomial variate with parameters n = 4 p $= \frac{1}{4}$ such that

P(X = r) = Probability that r balls are white $= ^4C_r\left(\frac{1}{4}\right)^r\left(\frac{3}{4}\right)^{4-r}$; r = 0, 1, 2, 3, 4 ...(i)

Now,

i. Probability that all are white = P(X = 4) $= {}^4C_4\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^{4-4} = \left(\frac{1}{4}\right)^4$ [Using (i)]
ii. Probability that only 3 are white = P(X = 3) $= {}^4C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^{4-3} = 3\left(\frac{1}{4}\right)^3$ [Using (i)]
iii. Probability that none is white = P(X = 0) $= {}^4C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^4 = \left(\frac{3}{4}\right)^4$ [Using (i)]
iv. Probability that at least three are white = P(X $\geq$ 3) = P(X = 3) + P(X = 4)
$= {}^4C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^{4-3} + {}^4C_4\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^0$ [Using (i)]
$= 13\left(\frac{1}{4}\right)^4$

OR

Probability that α student graduates = 0.4

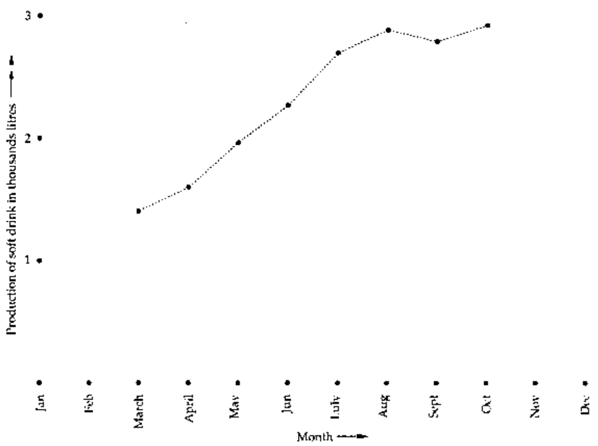
i. Probability that none of 3 students graduate = (1 - 0.4) $\times$ (1 - 0.4) $\times$ (1 - 0.4)
Probability that student (1) doesn't graduate
$\Rightarrow$ 0.6 $\times$ 0.6 $\times$ 0.6
$\Rightarrow$ 0.216
$\therefore$ Probability that none of 3 students graduate = 0.216
ii. The only one will graduate.
we select one of 3 in $^3C_1$ ways his probability of passing = 0.4 remaining two fail
$\Rightarrow$ 0.6 $\times$ 0.6
probability = $^3C_1$ $\times$ (0.4) $\times$ (0.6) $\times$ (0.6)
= 3 $\times$ 0.4 $\times$ 0.36
Probability that only one will graduate
$\Rightarrow$ 0.432
iii. Probability that all students graduate = (0.4)(0.4)(0.4) = 0.064
$\therefore$ Probability that all will graduate = 0.064

30. Calculation of five monthly moving averages:

Month	**Production of soft drink (in thousands litres)**	**5 monthly totals**	**5 monthly moving averages**
January	1.2	-	-
February	0.8	-	-
March	1.4	6.8	1.36
April	1.6	8.0	1.6
May	1.8	9.8	1.96

June	2.4	11.4	2.28
July	2.6	13.4	2.68
August	3.0	14.4	2.88
September	3.6	13.9	2.78
October	2.8	14.7	2.94
November	1.9	-	-
December	3.4	-	-

These moving averages are plotted on the following graph:



31. It is given that:

μ = Population mean = 0.7, $\bar{X}$ = Sample mean = 0.742

n = Sample size = 10 and, s = Sample standard deviation = 0.04

We define,

Null Hypothesis H_0: There is no significant difference between the sample mean $\bar{X}$ and the population mean μ or, the product is not inferior. Alternate hypothesis H_1: The difference between the sample mean $\bar{X}$ and the population mean μ is. significant i.e. $\mu \neq \bar{X}$ or the product is inferior.

Let t be the test statistic given by

t = $\frac{\bar{X}-\mu}{\frac{s}{\sqrt{n-1}}}$ or, t = $\frac{(\bar{X}-\mu)\sqrt{n-1}}{s}$

$\Rightarrow t = \frac{(0.742-0.7)}{0.04}\sqrt{10-1} = \frac{0.042}{0.04} \times 3 = \frac{126}{40}$ = 3.15

The test statistic 't' follows Student's t -distribution with (10 - 1) = 9 degrees of freedom. We shall now compare this calculated value with the tabulated value of t for 9 degrees of freedom and at a certain level of significance. It is given that $t_9(0.05)$ = 2.262

We observe that

|t| = 3.15 > 2.262 = $t_9(0.05)$

i.e. Calculated |t| > Tabulated $t_9(0.05)$

So, the null hypothesis is rejected at 5% level of significance or the alternative hypothesis is accepted at 5% level of significance. Hence, the sample mean $\bar{X}$ differs significantly from the population mean μ i.e. the work is inferior.

Section D

32. Converting the given inequations into equations, we obtain the following equations:

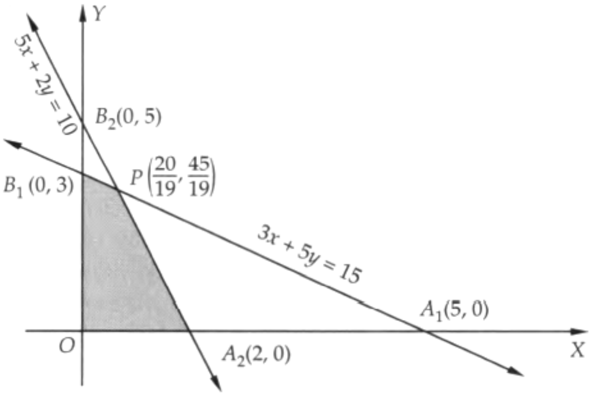
3x + 5y = 15, 5x + 2y = 10, x = 0 and y = 0

Region represented by 3x + 5y $\leq$ 15: The line 3x + 5y = 15 meets the coordinate axes at $A_1(5, 0)$ and $B_1(0, 3)$ respectively. Join these points to obtain the line 3x + 5y = 15. Clearly, (0,0) satisfies the inequation 3x + 5y $\leq$ 15. So, the region containing the origin represents the solution set of the inequation 3x + 5y < 15.

Region Represented by 5x + 2y $\leq$ 10: The line 5x + 2y = 10 meets the coordinate axes at $A_2(2, 0)$ and $B_2(0, 5)$ respectively. Join these points to obtain the graph of the line 5x + 2y = 10. Clearly, (0, 0) satisfies the inequation 5x + 2y $\leq$ 10. So, the region containing the origin represents the solution set of this inequation.

Region represented by x $\geq$ 0 and y $\geq$ 0: Since every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0 and y $\geq$ 0. The shaded region OA_2 PB_1 in figure represents the common region of the above inequations. This region is the feasible region of the given LPP.

The coordinates of the vertices (comer-points) of the shaded feasible region are O(0, 0), $A_2(2, 0)$, $P(\frac{20}{19}, \frac{45}{19})$ and $B_1(0, 3)$.



These points have been obtained by solving the equations of the corresponding intersecting lines, simultaneously.

The values of the objective function at these points are given in the following table:

Point (x, y)	**Value of the objective function Z = 5x + 3y**
O(0, 0)	Z = 5 × 0 + 3 × 0 = 0
$A_2(2, 0)$	Z = 5 × 2 + 3 × 0 = 10
$P\left(\frac{20}{19}, \frac{45}{19}\right)$	$Z = 5 \times \frac{20}{19} + 3 \times \frac{45}{19} = \frac{235}{19}$
$B_1(0, 3)$	Z = 5 × 0 + 3 × 3 = 9

Clearly, Z is maximum at $P\left(\frac{20}{19}, \frac{45}{19}\right)$. Hence, $x = \frac{20}{19}$, $y = \frac{45}{19}$ is the optimal solution of the given LPP and the optimal value of Z is $\frac{235}{19}$.

OR

Converting the given inequations into equations, we get

2x + 3y = 6, x - 2y = 2, 3x + 2y = 12, -3x + 2y = 3, x = 0 and y = 0

Region represented by $-2x - 3y \leq -6$: The line -2x - 3y = -6 or, 2x + 3y = 6 cuts OX and OY at $A_1(3, 0)$ and $B_1(0, 2)$ respectively.

Join these points to obtain the line 2x + 3y = 6.

Since O(0, 0) does not satisfy the inequation $-2x - 3y \leq -6$. So, the region represented by $-2x - 3y \leq -6$ is that part of the XOY-plane which does not contain the origin.

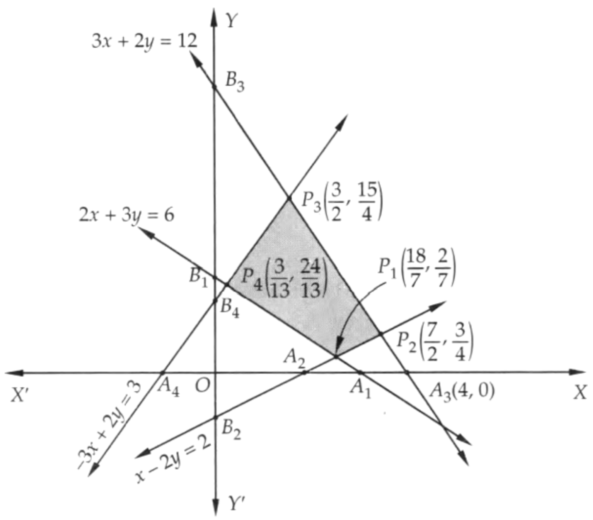
Region represented by $x - 2y \leq 2$: The line x - 2y = 2 meets the coordinate axes at $A_2(2, 0)$ and $B_2(0, -1)$. Join these points to obtain x - 2y = 2. Since (0, 0) satisfies the inequation $x - 2y \leq 2$, so the region containing the origin represents the solution set of this inequation.

Region represented by $3x + 2y \leq 12$: The line $3x + 2y \leq 12$ intersects OX and OY at $A_3(4, 0)$ and $B_3(0, 6)$. Join these points to obtain the line 3x + 2y =12. Clearly, (0, 0) satisfies the inequation $3x + 2y \leq 12$. So, the region containing the origin is the solution set of the given inequations.

Region represented by $-3x + 2y \leq 3$: The line -3x + 2y = 3 intersects OX and OY at $A_4(-1, 0)$ and $B_4(0, 3/2)$. Join these points to obtain the line -3x + 2y = 3. Clearly, (0, 0) satisfies this inequation. So, the region containing the origin represents the solution set of the given inequation.

The region represented by $x \geq 0$, $y \geq 0$: Clearly, the XOY quadrant represents the solution set of these two inequations.

The shaded region shown in the figure represents the common solution set of the above inequations. This region is the feasible region of the given LPP.



The coordinates of the corner points (vertices) of the shaded feasible region P_1 P_2 P_3 P_4 are $P_1\left(\frac{18}{7}, \frac{2}{7}\right)$, $P_2\left(\frac{7}{2}, \frac{3}{4}\right)$, $P_3\left(\frac{3}{2}, \frac{15}{4}\right)$ and $P_4\left(\frac{3}{13}, \frac{24}{13}\right)$. The points have been obtained by solving the equations of the corresponding intersecting lines, simultaneously.

The values of the objective function at these points are given in the following table:

Point (x, y)	**Value of the objective function Z = 5x + 2y**
$P_1\left(\frac{18}{7}, \frac{2}{7}\right)$	$Z = 5 \times \frac{18}{7} + 2 \times \frac{2}{7} = \frac{94}{7}$
$P_2\left(\frac{7}{2}, \frac{3}{4}\right)$	$Z = 5 \times \frac{7}{2} + 2 \times \frac{3}{4} = 19$
$P_3\left(\frac{3}{2}, \frac{15}{4}\right)$	$Z = 5 \times \frac{3}{2} + 2 \times \frac{15}{4} = 15$
$P_4\left(\frac{3}{13}, \frac{24}{13}\right)$	$Z = 5 \times \frac{3}{13} + 2 \times \frac{24}{13} = \frac{63}{13}$

Clearly, Z is minimum at $x = \frac{3}{13}$ and $y = \frac{24}{13}$ and maximum at $x = \frac{7}{2}$ and $y = \frac{3}{4}$. The minimum and maximum values of Z are $\frac{63}{13}$ and 19 respectively.

33. 3:1

34. Given p = 1% = 0.01, n = 100

So, $\lambda = 100 \times 0.01 \Rightarrow \lambda = 1$

$P(0) = e^{-\lambda} = e^{-1} = 0.368$

Now, we use the recurrence relation of the Poisson distribution

$P(r + 1) = \frac{\lambda}{r+1} P(r)$

$P(1) = \frac{\lambda}{1}\ P(0) = 1 \times 0.368 = 0.368$

$P(2) = \frac{\lambda}{2}\ P(1) = \frac{1}{2} \times 0.368 = 0.184$

$P(3) = \frac{\lambda}{3}\ P(2) = \frac{1}{3} \times 0.184 = 0.0613$

$P(4) = \frac{\lambda}{4}\ P(3) = \frac{1}{4} \times 0.0613 = 0.0153$

$P(5) = \frac{\lambda}{5}\ P(4) = \frac{1}{5} \times 0.0153 = 0.0031$

i. $P(X \geq 3) = 1 - \{P(0) + P(1) + P(2)\}$

$= 1 - (0.368 + 0.368 + 0.184)$

$= 1 - 0.92 = 0.08$

ii. Required probability = P(1) + P(2) + P(3)

$= 0.368 + 0.184 + 0.0613 = 0.6133$

iii. $P(X \leq 2) = P(0) + P(1) + P(2)$

$= 0.368 + 0.368 + 0.184 = 0.92$

OR

Given number of phone calls arrives per hour is 48.

i. Let random variable X be the number of phone calls in 5 minutes interval of time, then

$\lambda = 48 \times \frac{5}{60} \Rightarrow \lambda = 4$

So, $P(X = 3) = \frac{4^3 \cdot e^{-4}}{3!} = \frac{64 \times 0.018}{6} = 0.192$

ii. Let random variable X be the number of phone calls in 15 minutes interval of time, then

$\lambda = 48 \times \frac{15}{60} \Rightarrow \lambda = 12$

So, P(X = 10) = $\frac{12^{10}e^{-12}}{10!}$

iii. Let random variable X be the number of phone calls in 5 minutes interval of time, then

$\lambda = 48 \times \frac{5}{60} \Rightarrow \lambda = 4$

So, we expect that 4 callers will be waiting during that time.

Now, P(X = 0) = $\frac{4^0e^{-4}}{0!}$ = 0.018

iv. Let random variable X be the number of phone calls in 3 minutes interval of time, then

$\lambda = 48 \times \frac{3}{60} \Rightarrow \lambda = 2.4$

P(X = 0) = $\frac{2.4^0e^{-2.4}}{0!}$ = 0.091

Hence, the required probability is 0.091

35. i. Annual amount of depreciation $= \frac{45200-0}{3} = ₹\frac{45200}{3}$

Rate of depreciation $= \frac{\frac{45200}{3}}{45200} \times 100 = 33.3\%$

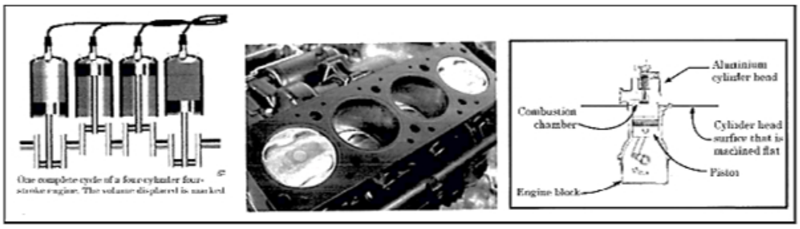
ii. v(t) = mt + C $-\frac{45200}{3}t + 45200$

iii. $v\left(1\frac{1}{2}\right) = -\frac{45200}{3} \times \frac{3}{2} + 45200 = ₹22600$

Section E

36. **Read the text carefully and answer the questions:**

Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore



The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area 75π cm^2.

(i) $\pi r^2 + 2\pi rh = 75\pi \Rightarrow h = \frac{75-r^2}{2r}, \therefore V = \pi r^2 h = \frac{\pi}{2}(75r - r^3)$

(ii) $\frac{dV}{dr} = \frac{\pi}{2}(75 - 3r^2)$

(iii) $\frac{dV}{dr} = 0 \Rightarrow r = 5, \left.\frac{d^2V}{dr^2}\right]_{r=5} = \frac{\pi}{2}(-6r) < 0 \therefore$ volume is maximum when r = 5

OR

False,

$\frac{dV}{dr} = 0 \Rightarrow r = 5, \left.\frac{d^2V}{dr^2}\right]_{r=5} = \frac{\pi}{2}(-6r) < 0 \therefore$ volume is maximum when r = 5

As volume is maximum at r = 5 $\Rightarrow$ h = $\frac{75-5^2}{2(5)}$ = 5 $\Rightarrow$ h = r

37. **Read the text carefully and answer the questions:**

Loans are an integral part of our lives today. We take loans for a specific purpose - for buying a home, or a car, or sending kids abroad for education - loans help us achieve some significant life goals. That said, when we talk about loans, the word “EMI", eventually crops up because the amount we borrow has to be returned to the lender with interest.

Suppose a person borrows ₹1 lakh for one year at the fixed rate of 9.5 percent per annum with a monthly rest. In this case, the EMI for the borrower for 12 months works out to approximately ₹8,768.

Example:

In year 2000, Mr. Tanwar took a home loan of ₹3000000 from State Bank of India at 7.5% p.a. compounded monthly for 20 years.

(i) ₹ 24167.82

(ii) ₹ 10458.69

(iii) ₹ 13709.13

OR

₹ 410293.41

38. We have, A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$

$\therefore$ |A| = $\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$ = 1(1 + 3) + 1(2 + 3) + 1(2 - 1) = 10 $\neq$ 0

So, A is invertible.

Let C_{ij} be the co-factors of elements a_{ij} in A [a_{ij}]. Then,

$C_{11} = (-1)^{1+1}\begin{vmatrix} 1 & -3 \\ 1 & 1 \end{vmatrix} = 4$, $C_{12} = (-1)^{1+2}\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = -5$,

$C_{13} = (-1)^{1+3}\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1$, $C_{21} = (-1)^{2+1}\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = 2$,

$C_{22} = (-1)^{3+1}\begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} = 2$, $C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = -2$,

$C_{31} = (-1)^{3+1}\begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} = 2$, $C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = -2$,

and, $C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 3$

$\therefore$ adj A = $\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}^T = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

$\Rightarrow A^{-1} = \frac{1}{|A|}$ adj A = $\frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ ....(i)

Now, the given system of equations is expressible as

$$\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

or, $A^TX = B$, where X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

Now, $|A^T| = |A| = 10 \neq 0$. So, the given equations is consistent with a unique solution given by

$X = (A^T)^{-1} B = (A^{-1})^T B$ [$\because (A^T)^{-1}) = (A^{-1})^T$]

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}^T\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} \text{ [Using (i)]}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16+0+2 \\ 8+0-4 \\ 8+0+6 \end{bmatrix} = \begin{bmatrix} 9/5 \\ 2/5 \\ 7/5 \end{bmatrix}$$

$\Rightarrow x = \frac{9}{5}$, $y = \frac{2}{5}$ and $z = \frac{7}{5}$

Hence, $x = \frac{9}{5}$, $y = \frac{2}{5}$, $z = \frac{7}{5}$ is the required solution.

OR

Here,

A = $\begin{bmatrix} 3-3 & 4 \\ 2-3 & 4 \\ 0-1 & 1 \end{bmatrix}$

|A| = 3 (-3 + 4) + 3(2 - 0) + 4 (-2 - 0)

= 3 + 6 - 8 = 1

Co-factors of A are:

$C_{11} = 1$, $C_{21} = -1$, $C_{31} = 0$

$C_{12} = -2$, $C_{22} = 3$, $C_{32} = -4$

$C_{13} = -2$, $C_{23} = 3$, $C_{33} = -3$

adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}^T$

So, adj A = $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

Also, $A^2 = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

= $\begin{bmatrix} 9-6+0 & -9+9-4 & 12-12+4 \\ 6-6+0 & -6+9-4 & 8-12+4 \\ 0-2+0 & 0+3-1 & 0-4+1 \end{bmatrix}$

= $\begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$

$A^3 = A^2A = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

= $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$ = A^{-1}

Hence, $A^{-1} = A^3$